Nested parallelism in Accelerate

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GPUs

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  - This one: 2688 cores @ 867 MHz

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- How can we take advantage of this power?

With a high-level embedded language of course!

Accelerate

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- Data parallel language operating over multi-dimensional arrays

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```
dotp :: Acc (Vector Float) -> Acc (Vector Float) -> Acc (Scalar Float)
dotp xs ys = fold (+) 0 (zipWith (*) xs ys)
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Accelerate

- Data parallel language operating over multi-dimensional arrays

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type Vector e = Array (Z:.Int) e
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```
zipWith :: (Exp a -> Exp b -> Exp c)
        -> Acc (Array sh a)
        -> Acc (Array sh b)
        -> Acc (Array sh c)
```

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dotp :: Acc (Vector Float) -> Acc (Vector Float) -> Acc (Scalar Float)
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```
fold :: (Exp e -> Exp e -> Exp e)
     -> Exp e
     -> Acc (Array (sh:.Int) e)
     -> Acc (Array sh e)
```

```
zipWith :: (Exp a -> Exp b -> Exp c)
        -> Acc (Array sh a)
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A (slightly) more complex example

- Matrix-vector multiplication.

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```
mvm :: Acc (Array (Z:.Int:.Int) Float)
    -> Acc (Vector Float)
    -> Acc (Vector Float)
mvm mat vec = generate (index1 (height mat))
                       (λi -> the (dotp vec (getRow i mat)))
```

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index1 :: Exp Int -> Exp (Z:.Int)
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```
*** Exception: Cyclic definition of a value of type 'Exp' (sa = 46)
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Nested parallelism

- The “right” way

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

1	2	3	4	5

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mvm :: Acc (Array (Z:.Int:.Int) Float)
    -> Acc (Vector Float)
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  =
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```
mvm :: Acc (Array (Z:.Int:.Int) Float)
    -> Acc (Vector Float)
    -> Acc (Vector Float)
mvm mat vec
  =                             replicateRow (height mat) vec
```

- The “right” way

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

×

1	2	3	4	5
1	2	3	4	5
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```
mvm :: Acc (Array (Z:.Int:.Int) Float)
    -> Acc (Vector Float)
    -> Acc (Vector Float)
mvm mat vec
  =             zipWith (*) mat (replicateRow (height mat) vec)
```

- The “right” way

1	4	9	16	25
6	14	24	36	50
11	24	29	56	75
16	34	54	76	100

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- The “right” way

55
130
195
280

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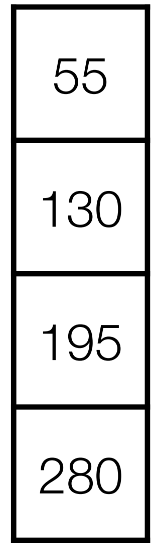


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We want to write the first version

Enabling nested parallelism

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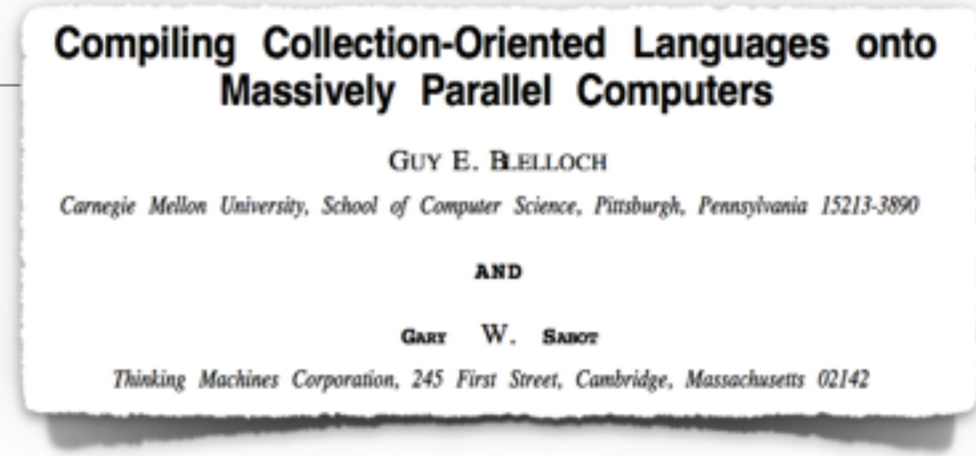

Compiling Collection-Oriented Languages onto Massively Parallel Computers

GUY E. BLELLOCH

Carnegie Mellon University, School of Computer Science, Pittsburgh, Pennsylvania 15213-3890

AND

GARY W. SABOT

Thinking Machines Corporation, 245 First Street, Cambridge, Massachusetts 02142

Enabling nested parallelism

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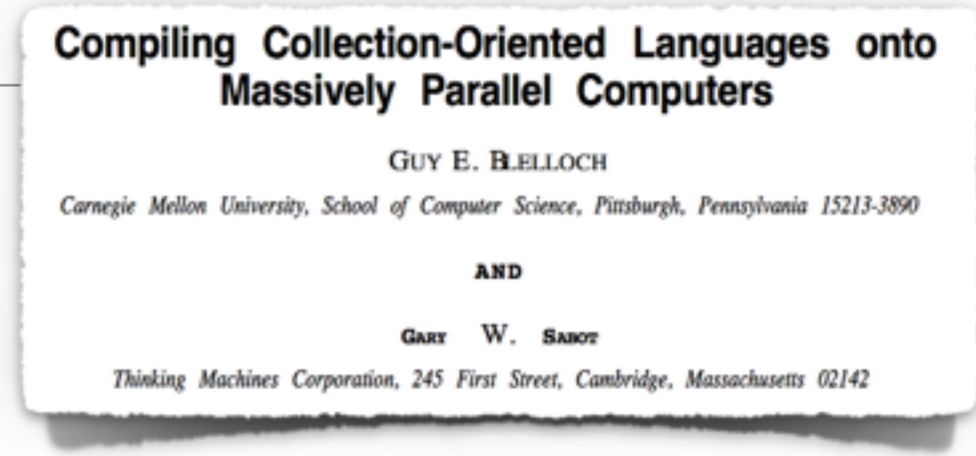

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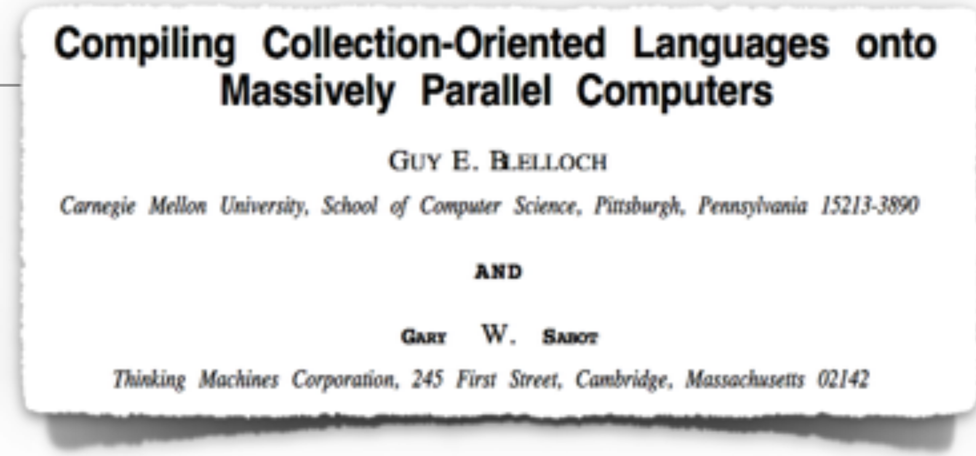

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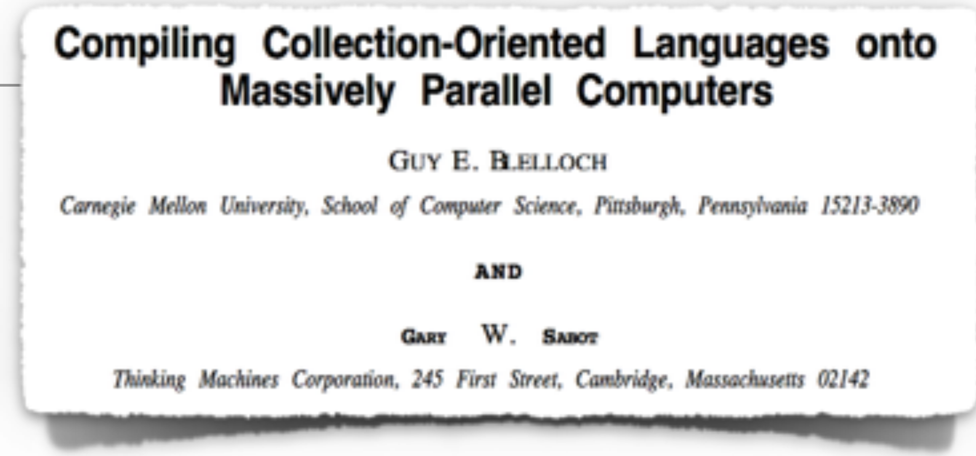

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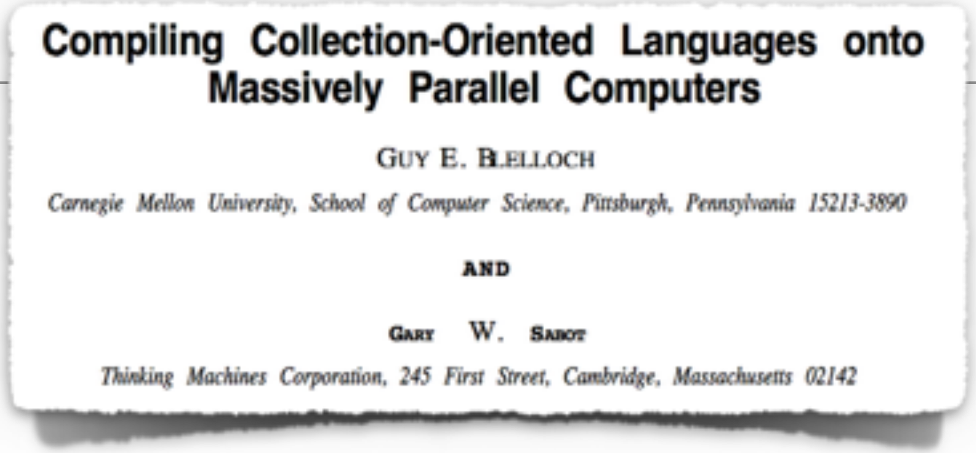

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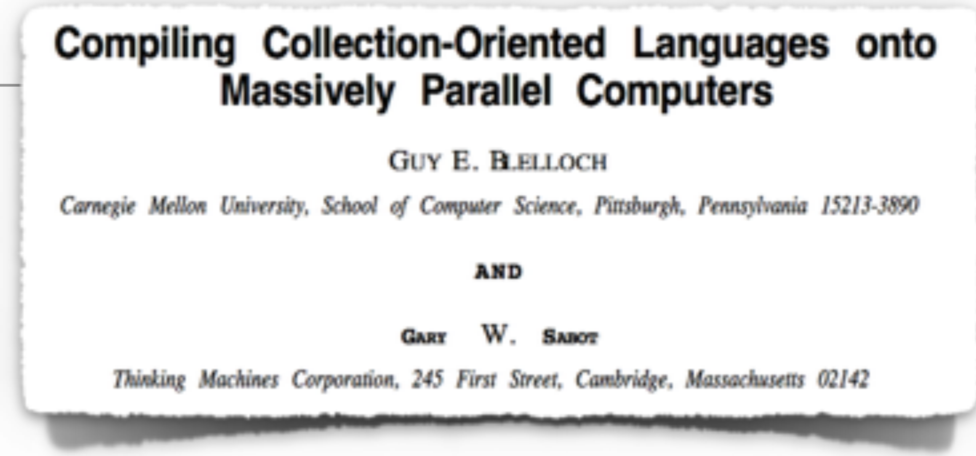
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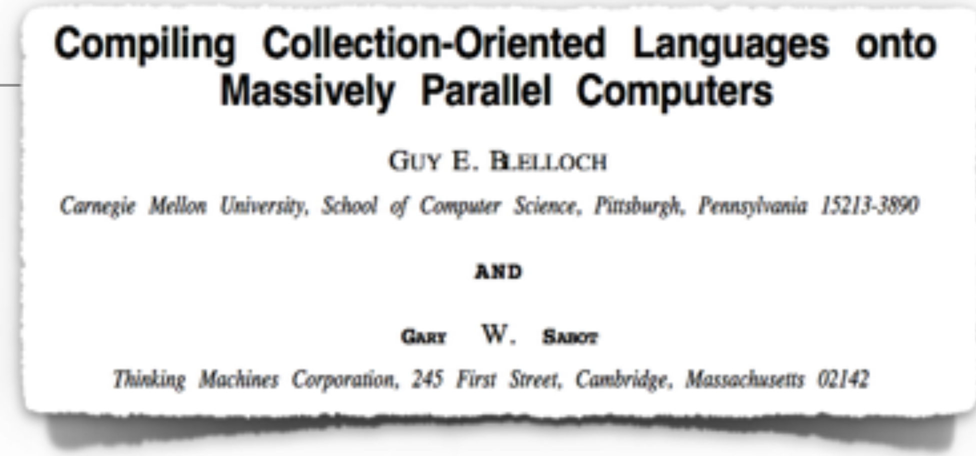

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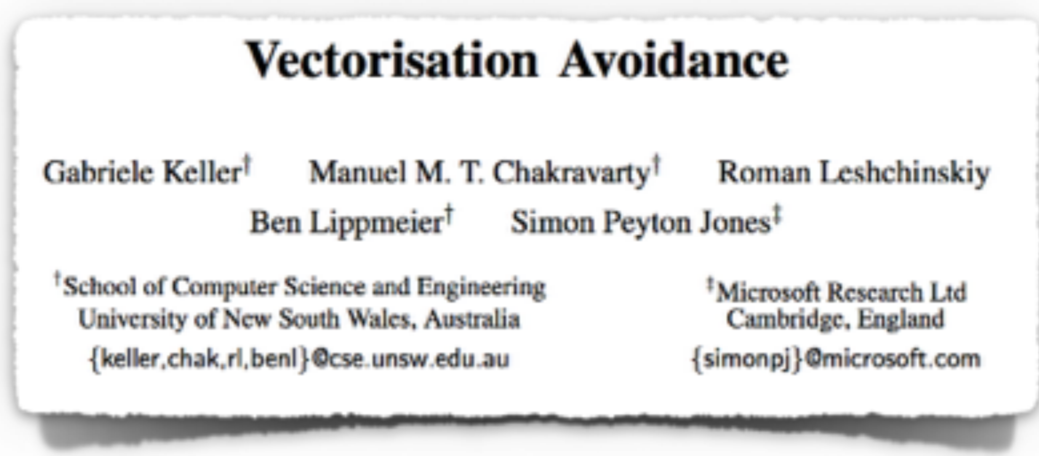

Vectorisation Avoidance

Gabriele Keller[†] Manuel M. T. Chakravarty[†] Roman Leshchinskiy
Ben Lippmeier[†] Simon Peyton Jones[‡]

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University of New South Wales, Australia
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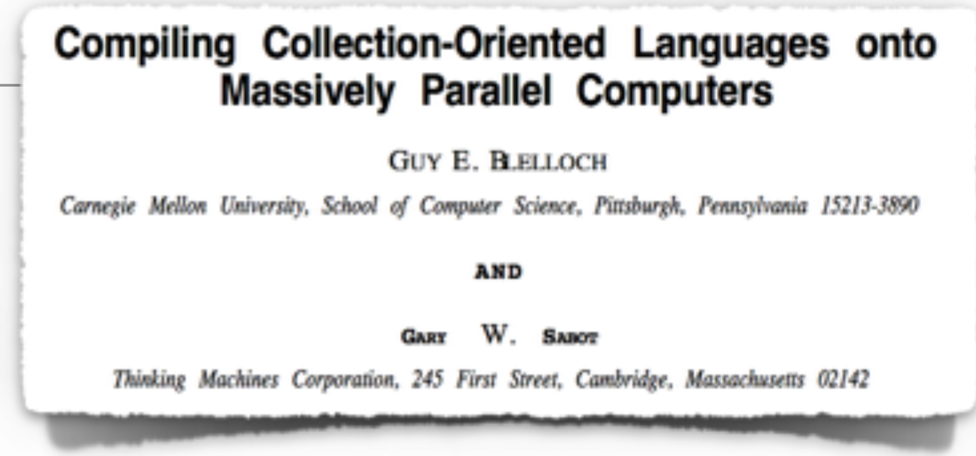
Enabling nested parallelism



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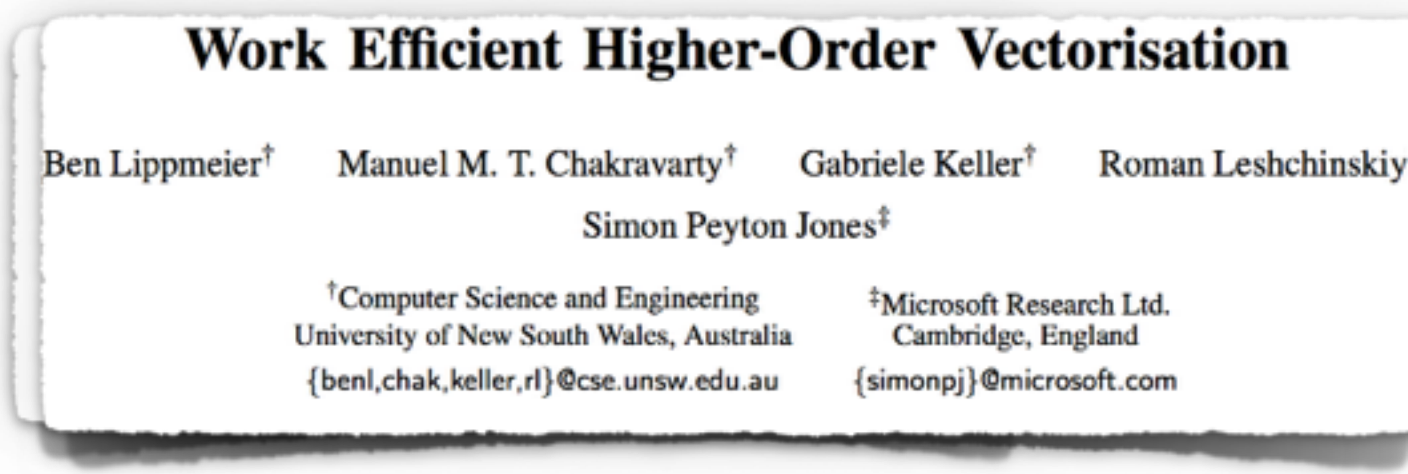

Work Efficient Higher-Order Vectorisation

Ben Lippmeier[†] Manuel M. T. Chakravarty[†] Gabriele Keller[†] Roman Leshchinskiy

Simon Peyton Jones[‡]

[†]Computer Science and Engineering
University of New South Wales, Australia
{benl,chak,keller,rl}@cse.unsw.edu.au

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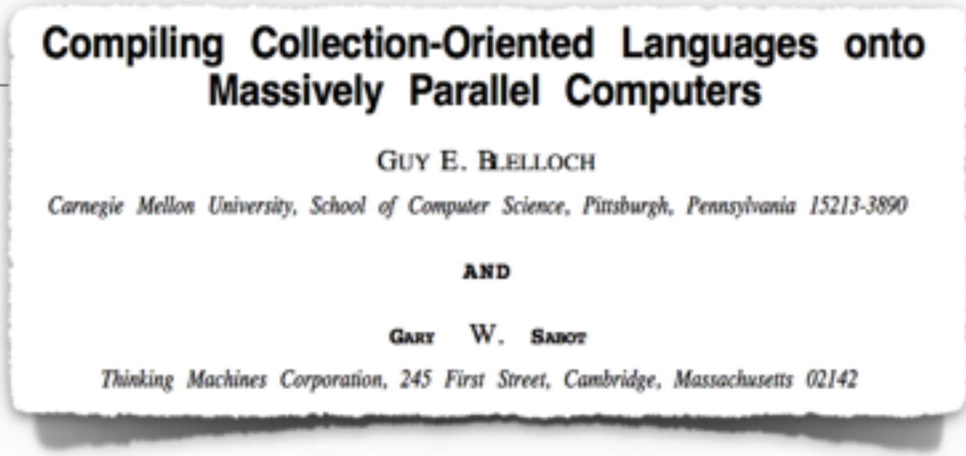

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Data Flow Fusion with Series Expressions in Haskell

Ben Lippmeier[†] Manuel M. T. Chakravarty[†] Gabriele Keller[†] Amos Robinson[†]

[†]Computer Science and Engineering
University of New South Wales, Australia
{benl,chak,keller,amosr}@cse.unsw.edu.au

The lifting transformation

```
foo     :: Int -> Float -> Float
```

$\mathcal{L}_n[\![$`foo`$]\!]$ `:: Vector Int -> Vector Float -> Vector Float`

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The expression being transformed

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The size parameter

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(Where c is a constant)

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The expression being transformed

$\mathcal{L}_n$ (Where c is a constant)

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The lifting transformation

$$\texttt{foo} \quad \texttt{:: Int -> Float -> Float}$$

$$\mathcal{L}_n[\![\texttt{foo}]\!] \quad \texttt{:: Vector Int -> Vector Float -> Vector Float}$$

The size parameter

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$\mathcal{L}_n$

$\mathcal{L}_n$ (Where
lifted equivalent)

The lifting transformation

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bar :: Int -> Int
bar = λx. 2*x + 1
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$\mathcal{L}_n[\![\texttt{bar}]\!]$ `:: Vector Int -> Vector Int`

$\mathcal{L}_n[\![\texttt{bar}]\!]$ `= λx. (replicate (length x) 2)` $*^{\uparrow}$ `x` $+^{\uparrow}$ `(replicate (length x) 1)`

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```
sum :: Vector Int -> Int
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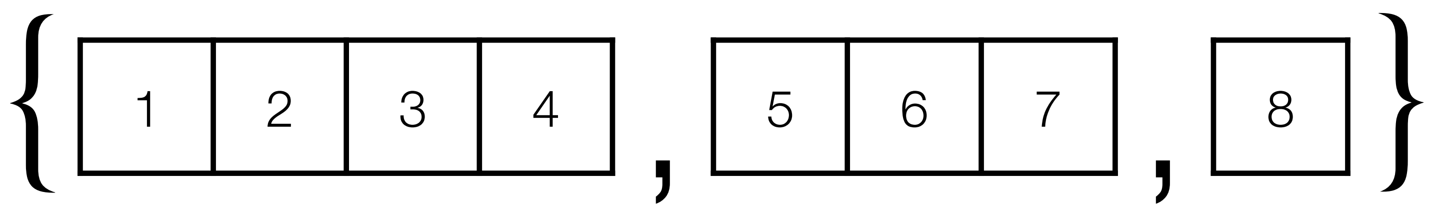

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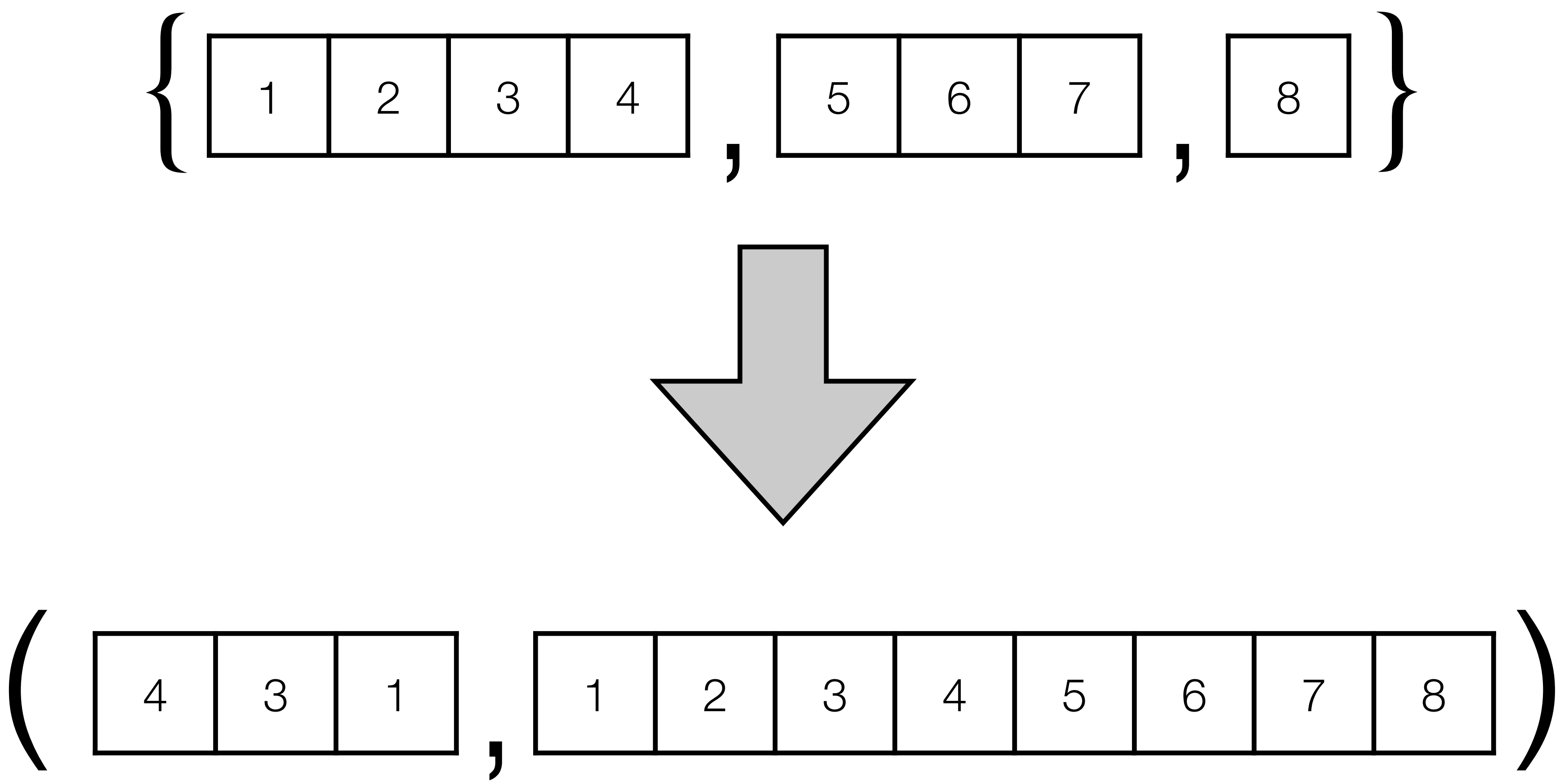

Vectorisation with multidimensional arrays

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Vectorisation with multidimensional arrays

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- Does it now require this?

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Vectors of arrays

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type Vector' = …a vector of arrays…
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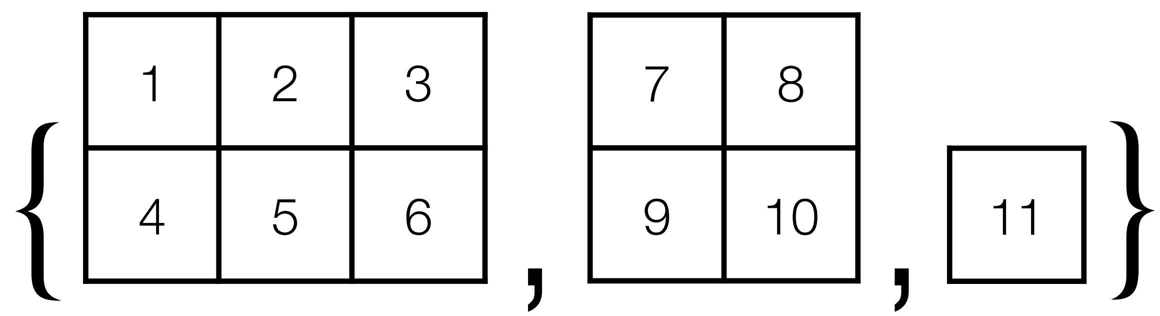
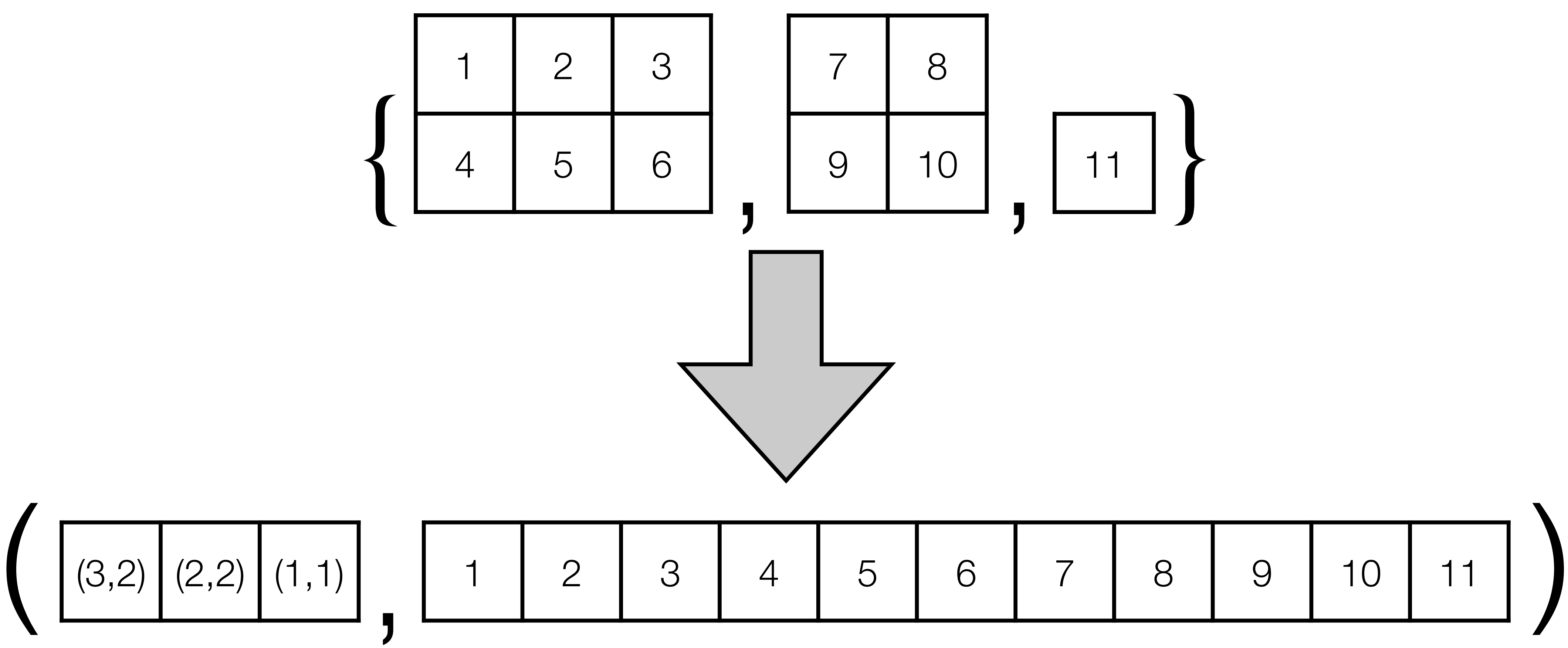
type Vector' = …a vector of arrays…
```



Vectors of arrays

```
type Vector' = …a vector of arrays…
```



Back to mvm

```
mvm mat vec = generate (index1 (height mat))
                       (λi -> the (dotp vec (getRow i mat)))
```

Back to mvm

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mvm mat vec = generate (index1 (height mat))
                       (λi -> the (dotp vec (getRow i mat)))

mvm mat vec
  = ℒn⟦(λi -> the (dotp vec (getRow i mat)))⟧
```

Back to mvm

```
mvm mat vec = generate (index1 (height mat))
                       (λi -> the (dotp vec (getRow i mat)))

mvm mat vec
  = ℒn⟦(λi -> the (dotp vec (getRow i mat)))⟧
    (map index1 [0..height mat])
```

Back to mvm

```
mvm mat vec = generate (index1 (height mat))
                       (λi -> the (dotp vec (getRow i mat)))
mvm mat vec
  = ℒn⟦(λi -> the (dotp vec (getRow i mat)))⟧
      (map index1 [0..height mat])

mvm mat vec
  = (λi -> the↑ (ℒn⟦dotp⟧ (replicate (length i) vec)
                          (getRow↑ i (replicate (length i) mat)))
      (map index1 [0..height mat])
```

```
mvm mat vec
  = (λi -> the↑ (ℒn⟦dotp⟧ (replicate (length i) vec)
                           (getRow↑ i (replicate (length i) mat))))
    (map index1 [0..height mat])
```

```
mvm mat vec
  = (λi -> the↑ (ℒn⟦dotp⟧ (replicate (length i) vec)
                          (getRow↑ i (replicate (length i) mat))))
    (map index1 [0..height mat])
```

Inlining, Fusion, simplification, etc.

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  = (λi -> the↑ (ℒn⟦dotp⟧ (replicate (length i) vec)
                             (getRow↑ i (replicate (length i) mat))))
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```

Inlining, Fusion, simplification, etc.

```
mvm mat vec = snd $ fold↑ (+) 0 (let h = height mat
                                 in ( replicate h (shape vec)
                                    , zipWith (*)
                                              (replicate↑ h vec)
                                              (flatten mat)))
```

```
mvm mat vec
  = (λi -> the↑ (ℒn⟦dotp⟧ (replicate (length i) vec)
                           (getRow↑ i (replicate (length i) mat))))
    (map index1 [0..height mat])
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simplification, etc.

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```

A vector of arrays

```
mvm mat vec
  = (λi -> the↑ (ℒn⟦dotp⟧ (replicate (length i) vec)
                           (getRow↑ i (replicate (length i) mat))))
    (map index1 [0..height mat])
```

Inlining, Fusion, simplification, etc.

```
mvm mat vec = snd $ fold↑ (+) 0 (let h = height mat
                                  in ( replicate h (shape vec)
                                     , zipWith (*)
                                               (replicate↑ h vec)
                                               (flatten mat)))
```

A vector of arrays
… but they're all the same size?

```
mvm mat vec
  = (λi -> the↑ (ℒn⟦dotp⟧ (replicate (length i) vec)
                          (getRow↑ i (replicate (length i) mat))))
    (map index1 [0..height mat])
```

Inlining, Fusion, simplification, etc.

```
mvm mat vec = snd $ fold↑ (+) 0 (let h = height mat
                                 in ( replicate h (shape vec)
                                    , zipWith (*)
                                              (replicate↑ h vec)
                                              (flatten mat)))
```

Replace with normal fold?

A vector of arrays
… but they're all the same size?

?

```
mvm mat vec
  = fold (+) 0 (zipWith (*) mat (replicateRow (height mat) vec))
```

```
mvm mat vec = snd $ fold↑ (+) 0 (let h = height mat
                                 in ( replicate h (shape vec)
                                    , zipWith (*)
                                              (replicate↑ h vec)
                                              (flatten mat)))
```

?

```
mvm mat vec
  = fold (+) 0 (zipWith (*) mat (replicateRow (height mat) vec))
```

Questions?